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Focal Law Calculations for Annular Phased Array Transducers

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Abstract
One of the most important characteristics of annular arrays is their superior focal capabilities. In order to achieve the desired focussing, each of the elements is fired in different times. The delay times used for beam focussing are called focal laws. The focal laws are automatically calculated from the inspection software. In order to make sure that the focal laws are calculated correctly from the software, it is wise to validate the focal laws for a particular configuration. The validation/comparison will involve the calculation of focal law values using three different methods, from the experimental acquisition software (TWI software) and from simulation software (CIVA). Moreover, one of the methods introduces the concept of centre of mass as an exit point of the beam, which is novel to be investigated because previously only the geometric centre was considered for annular arrays.

Keywords: Ultrasonic Testing (UT), Annular Phased Array Transducers, Focal Law Calculations, Centre of Mass, Focussing

1. Introduction

Due to some inherent limitations, e.g. no beam steering, annular array probes are not used as widely as linear array probes. Despite this, annular probes have a lot of unique characteristics which make them superior for some applications. Due to their unique geometry they are able to focus at great depths into a material with both a symmetric and circular focal point [1]. Because of the lack of knowledge that exists for this unique type of transducer, the investigation has had to start from the fundamentals. The approach adopted is the same as that followed for near field distance calculation in which a novel equation was generated [2].

2. Overall Methodology and Results

2.1 Equations and Methodology

Normally, for one medium, focal laws are easily calculated using simple trigonometric rules. But, as the probes used in this study are immersion probes the additional water paths must be accommodated in the calculations. The problem with the two mediums is that they have different velocities and the index point between the mediums is hard to find.

Three different methods are used for the calculation of focal laws. The simplest method, Method 1, involves the transformation of one medium to the other. In other words, the thickness of medium 1 will be virtually transformed to the equivalent medium 2 by multiplying it with the ratio of the two velocities. The mathematics behind this method are explained below. Equation 1 is the final equation for Method 1 in order to calculate the focal laws for each element. This method uses the centre of the element as the exit point of the beam.

$$T.W = F \cdot \frac{V_T}{V_W} + W.P$$

Where: **T.W** is the virtual total water path
F is the focus depth
$\mathbf{V_T}$ is the velocity of titanium
$\mathbf{V_W}$ is the velocity of water
W.P is the water path

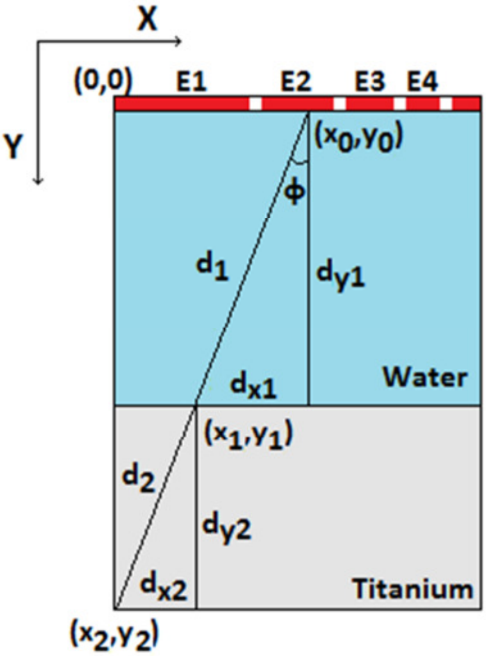
$$d_n = \sqrt{\left(\frac{D_n + E.W_n}{2}\right)^2 + T.W^2}$$

Where: $\mathbf{d_n}$ is the total travelled distance in virtual water
$\mathbf{D_n}$ is the internal diameter of the element
$\mathbf{E.W_n}$ is the element width
T.W is the virtual total water path

$$t_n = \frac{d_{ex} - d_n}{V_w} \quad (1)$$

Where: $\mathbf{t_n}$ is the focal law
$\mathbf{d_{ex}}$ is the total travelled distance of the outer element
$\mathbf{d_n}$ is the total travelled distance for particular element
$\mathbf{V_W}$ is the velocity of water

The second method, Method 2, was derived using Fermat's principle which states that the sound beam travels the shortest distance in a medium. This method does not involve any particular assumption like the previous method and it is applicable in two mediums directly. Figure 1 demonstrates the main principles for Method 2. The difficult part is to define the x_1 point. This is accomplished by using the calculation tool of Excel which finds this point with respect to the smallest value of TOF (Fermat's principle). Just like Method 1, Method 2 is using the centre of the element as the exit point of the beam. Equation 2 is the final equation of Method 2 in order to calculate the focal laws for each element.



$$d_{x1} = x_0 - x_1$$
$$d_{y1} = y_0 - y_1$$
$$d_{x2} = x_1 - x_2$$
$$d_{y1} = y_1 - y_2$$
$$d_1 = \sqrt{d_{x1}^2 + d_{y1}^2}$$
$$d_2 = \sqrt{d_{x2}^2 + d_{y2}^2}$$

Figure 1. Sketch of the main principles of Method 2

$$TOF_n = \frac{d_1}{V_W} + \frac{d_2}{V_T}$$

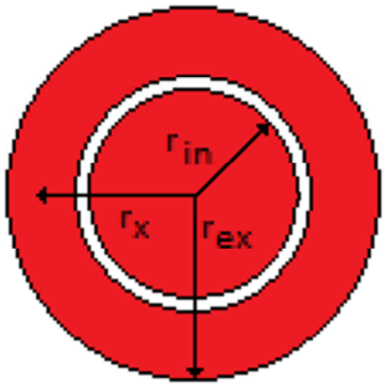
Where: $\mathbf{TOF_n}$ is the total time of flight that the beam travels in both mediums
$\mathbf{d_1}$ is the distance that the beam travels in water for particular element
$\mathbf{d_2}$ is the distance that the beam travels in titanium for particular element
$\mathbf{V_W}$ is the velocity of water
$\mathbf{V_T}$ is the velocity of titanium

$$t_n = TOF_{ex} - TOF_n (2)$$

Where: $\mathbf{t_n}$ is the focal law
$\mathbf{TOF_{ex}}$ is the total time of flight of the outer element
$\mathbf{TOF_n}$ is the total time of flight for particular element

The third method, Method 3, uses exactly the same principles and equations as Method 2 but instead of using the centre of the element as an exit point it uses the centre of mass. The theory of the centre of mass was originally generated when asymmetrical elements are used. The centre of mass is considered to be the point that separates the element into two different parts which have equal areas. For symmetric elements, the centre of the element coincides with the centre of mass. But on asymmetrical elements the centre of mass is at a different point from the centre of the element. Since the ring is an asymmetric shape, it is worth investigating this parameter and present any differences. It is worth mentioning that the whole theory of the centre of mass as an exit point was not completely validated and further investigation is ongoing. Nevertheless, it is worth comparing it with the conventional theory.

Figure 2 shows the calculation of the centre of mass for ring elements. The values of the centre of mass are used to calculate the focal laws just like in Method 2. The only part that changes is the exit point of the element which is denoted as x_0 in Figure 1. All the other equations remain the same as Method 2.



$$\pi {r_x}^2 = \pi {r_{in}}^2 + \pi\left(\frac{{r_{ex}}^2 - {r_{in}}^2}{2}\right) \Leftrightarrow$$

$$ {r_x}^2 = {r_{in}}^2 + \frac{{r_{ex}}^2}{2} - \frac{{r_{in}}^2}{2} \Leftrightarrow$$

$$ r_x = \sqrt{\frac{{r_{ex}}^2 + {r_{in}}^2}{2}}$$

Where: $\mathbf{r_x}$ is the radius of the center of mass

$\mathbf{r_{in}}$ is the internal radius of the ring

$\mathbf{r_{ex}}$ is the external radius of the ring

Figure 2. Centre of mass sketch and equation derivation

2.2 Results

In this section, the focal law values calculated from the three different methods are compared as well as further comparison with the focal values generated from two software applications, TWI software, which is currently under development and CIVA. To summarise, the different methods of determining the calculated delay values are:

- Method 1 involves the transformation of one medium to the other using the proportionality of the velocities. The final equation that is used for this method is Equation 1. The centre of the element is considered as the exit point of the beam.

- Method 2 derived by Fermat's principle and the final equation that is used for the calculation of focal laws is Equation 2. The centre of the element is considered as the exit point of the beam.

- Method 3 uses the same equations and principle as Method 2 but the centre of mass is considered as the exit point of the beam.

The parameters for the focal law calculations used for all methods are provided in Table 1. Calculations were carried out for both annular probes and for the full aperture size. The focal law values are demonstrated in Table 2 and Table 3 for 2.5MHz and 5MHz probes, respectively.

Table 1. Parameters for the focal law calculations

Focus depth (mm)	50
Water path (mm)	70
Velocity of titanium (m/sec)	6100
Velocity of water (m/sec)	1480

Table 2. Focal laws for 2.5MHz probe

2.5 MHz probe	Software (µsec)		Calculations (µsec)		
Elements	Civa	TWI software	Method 1	Method 2	Method 3
1	0.680663	0.680714824	0.6956415	0.6806629	0.6808271
2	0.6135382	0.611823807	0.6283724	0.6135381	0.61190434
3	0.5647432	0.563814993	0.5792911	0.5647431	0.56387428
4	0.5155996	0.514975871	0.5297042	0.5155995	0.51504177
5	0.46589172	0.465457511	0.4793896	0.4658916	0.46550242
6	0.4157059	0.415365402	0.4284292	0.4157058	0.4154266
7	0.36507028	0.364768124	0.3768467	0.3650702	0.36486226
8	0.3139165	0.313718992	0.3245672	0.3139164	0.31376107
9	0.26240513	0.262264676	0.2717503	0.262405	0.26229242
10	0.21059509	0.210435307	0.2184526	0.210595	0.21051396
11	0.15837868	0.158277481	0.1645592	0.1583786	0.15832202
12	0.10588714	0.105796178	0.1102018	0.105887	0.1058535
13	0.05305122	0.053033072	0.0553051	0.0530511	0.0530325
14	0	0	0	0	0

Table 3. Focal laws for 5MHz probe

5 MHz probe	Software (µsec)		Calculations (µsec)		
Elements	Civa	TWI software	Method 1	Method 2	Method 3
1	0.38536334	0.385472038	0.390143	0.3853633	0.38542363
2	0.35480347	0.354123992	0.3595532	0.3548034	0.35408228
3	0.33207712	0.33171759	0.3367658	0.3320771	0.33169117
4	0.30890548	0.308658572	0.3134979	0.3089055	0.30865536
5	0.2852114	0.285049573	0.2896696	0.2852114	0.28503374
6	0.26107156	0.26095248	0.265356	0.2610715	0.26094256
7	0.23651515	0.236414803	0.2405843	0.2365151	0.23641628
8	0.21152233	0.211475296	0.2153325	0.2115223	0.21144693
9	0.18617648	0.186157007	0.1896829	0.1861764	0.1861189
10	0.16053998	0.160485298	0.1636969	0.1605399	0.16049719
11	0.13451557	0.134484633	0.1372742	0.1345155	0.13448156
12	0.10819545	0.108164096	0.1105067	0.1081954	0.10817303
13	0.0816466	0.081541531	0.0834611	0.0816466	0.0816305
14	0.0547149	0.054633834	0.0559788	0.0547149	0.05470486
15	0.02750212	0.02745229	0.0281617	0.0275021	0.02749789
16	0	0	0	0	0

3. Discussion

The main reason to calculate the focal laws was to validate the values of the acquisition software (developed by TWI) and simulation software (CIVA). These two software packages were used for experimental and computational work so it is vital to validate the automatically generated focal laws to ensure the accuracy of any results obtained.

The comparison of the values of CIVA and TWI software has shown that there is a minor difference between the focal laws. The differences are larger for the elements that are closer to the centre and minimal for the outer elements. For some elements the differences have been largely minimised or eliminated. In order to understand the differences, the obtained values from the two software packages must be compared with the calculated values. Method 1 gives good results that are close but they cannot be considered negligible. It is worth to reiterate that this method is not directly applicable for a two medium problem and requires the use of the ratio of the velocities for the two mediums. The limiting problem is that this method does not include the changing of the angle of index point between the two mediums which can make a significant difference. In this case the difference in the velocities is more than four times, and therefore, Method 1 is considered the least accurate method of all the methods investigated.

Methods 2 and 3 provide better results since they include the changing of the angle of the index point. As it can observed from Table 2 and Table 3 the calculated values from Method 2 are correlated exactly with the values from CIVA in the sixth decimal digit. This high accuracy provides confidence and allows to state that CIVA must consider the centre of the element as the exit point of the beam. On the other hand, the values from Method 3 are correlated with the values from TWI software. Therefore it can be stated with confidence that TWI software is considering the centre of mass as the exit point of the beam.

As far as the comparison between Method 2 and 3 is concerned, the differences between the values are decreased as the diameter of the element is increased. This phenomenon can be easily explained as the probes use the equal-area technique and the element width is decreased as the diameter of the element is increased. As the element width is decreased, the difference between the centre of mass and the centre of element is also decreased. For this reason, the focal law values follow the same trend and consequently the differences are negligible for the outer elements. Nevertheless, the results have shown that no further investigation is needed as far as the exit point of the beam is concerned. For an annular array configuration the differences are only significant for the inner elements and it is believed that even these differences are not large enough to have a major effect on the generated beams.

Furthermore, the validation of the CIVA and TWI software focal law calculations has been successfully performed and they can be considered as reliable.

4. Conclusions

Focal law calculations have concluded the following aspects:

- The validation of the focal laws values of acquisition software (TWI developed software) and simulation software (CIVA) have been made with the calculated values using three different computational methods.

- The comparison has demonstrated differences which were quantified by the different approach to exit points of the ultrasonic beam. Specifically, it was validated that CIVA is considering the centre of the element as the exit point of the beam and TWI software is considering the centre of mass as the exit point of the beam.

- The validation of the CIVA and TWI software’s focal law calculations was successful and the focal laws used can be considered as reliable.

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References

1. Bjorn A.J Angelsen, Hans Torp, Sverre Holm, Kjell Kristoffersen, T.A Whittingham (1995), Which transducer array is best? [Online]. January 1995. Available from: heim.ifi.uio.no/~sverre/papers/95_EurJUS.pdf [Accessed: 29/07/14]

2. Georgios Liaptsis, Dimosthenis Liaptsis, Peter Charlton (2014), Demonstration of a Novel Equation for the Near Field Distance Calculation for Annular Phased Array Transducers [Online]. November 2014. Available from: http://www.ndt.net/article/ndtnet/2014/95_Charlton.pdf [Accessed: 14/11/14]