Demonstration of a Novel Equation for the Near Field Distance Calculation for Annular Phased Array Transducers

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Abstract
Annular phased array transducers are well known in the medical field, with their main use being in ophthalmology and dermatology, and have started to be employed in the NDT industry relatively recently. They have a unique geometry and can focus at great depths into a material with both a symmetric and circular focal point. This paper tries to broaden the existing knowledge of these unique types of transducers. The paper starts with the necessary fundamentals and provides near field calculations for two different annular array probe configurations. The importance of this work relates to the limitations of the conventional equation to annular arrays. A major contribution is the generation of a novel equation for near field calculation for annular arrays. Validation of the novel near field equation has been carried out computationally, for both individual elements as well as different aperture sizes.

Keywords: ultrasound, annular phased array, Fresnel zone, near field calculations

1. Introduction

The concept of the Fresnel Zone, more commonly referred to as the near field is central in wave propagation to many areas (radiology, optics, sound) but the concept remains the same. The zone construction was first proposed by Fresnel in 1818, which attempted to explain the diffraction phenomenon by using Huygens principle [1]. According to Krautkramer et al [2], there are differences of the sound pressure at different points of the sound field. Therefore, the individual sound pressure values of the waves cannot be simply added but the path differences must be taken into account. He described that if the path difference between two waves is less than half wavelength (λ/2) then all the reflected waves constructively interfere. Therefore the end of the near field is defined at this point.

The end of the near field for circular transducers is calculated from the conventional equation, the derivation which is demonstrated in the next section. However, there is not an equation that is applicable to annular arrays which consist of ring-shaped elements. The need for a novel equation that is applicable to annular arrays is considered vital since it is important to be able to calculate the focal capabilities of individual elements as well as different aperture sizes within the array.

Annular probes are not used as widely as linear array probes because of some limitations. No beam steering and the complex programming of these probes are some of these limitations. Despite this, annular probes have unique characteristics which make them superior for some applications. Annular arrays have unique geometry and they can focus at great depths into a material with both a symmetric and circular focal point [3].

2. Overall Methodology and Results

2.1 Conventional equation derivation

The near field distance is defined as the point that the two waves have difference half of wavelength, as illustrated in Figure 1. The conventional equation for a circular transducer is defined below.

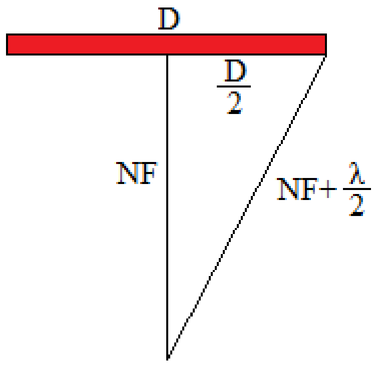

Figure 1. Demonstration of the near field (circular transducer)

$$\left(NF+\frac{\lambda}{2}\right)^2 = NF^2+\left(\frac{D}{2}\right)^2 \Leftrightarrow NF^2+2\cdot NF\cdot\frac{\lambda}{2}+\frac{\lambda^2}{4} = NF^2+\frac{D^2}{4} \Leftrightarrow$$

$$2\cdot NF\cdot\frac{\lambda}{2}=\frac{D^2}{4}-\frac{\lambda}{4} \Leftrightarrow NF=\frac{D^2}{4\cdot\lambda}-\frac{\lambda}{4}$$

Where: *NF* is the Near Field, *D* is the diameter of the transducer and λ is the wavelength. For simplification the $\lambda/4$ is usually ignored since it does not a make significant difference to the result. Therefore the conventional equations that are commonly used are:

$$NF=\frac{D^2}{4\cdot\lambda}\ (1)\ or\ NF=\frac{D^2\cdot f}{4\cdot V}\ (2)$$

Where: *NF* is the Near Field, *D* is the diameter of the transducer, λ is the wavelength, *f* is the frequency of the transducer and *V* is the velocity of the medium.

2.2 Novel equation derivation

Equations 1 and 2 use the external diameter of the transducer to calculate the near field. The challenge with the annular transducers is that they consist of ring-shaped elements instead of circular disk-shaped elements. Therefore, for individual elements, the diameter cannot be used because even though the element has a certain diameter, the element width is significantly smaller. For that reason, the hypothesis is to modify the existing near field equations and replace the diameter of the transducer with the area of the ring in the transducer. It is assumed that this is feasible as it can be derived by using simple algebra rules. The derivation for the new equation is presented below. Equation 3 is the final derived novel equation for near field.

$$A=\pi\cdot r^2 \Leftrightarrow A=\frac{\pi\cdot D^2}{4} \Leftrightarrow D^2=\frac{4\cdot A}{\pi}$$

Where: *A* is the area of the circle or ring, *r* is the radius and *D* is the diameter. If the diameter is substituted with the above equivalence then Equation 2 becomes:

$$NF=\frac{D^2\cdot f}{4\cdot V} \Leftrightarrow NF=\frac{4\cdot A\cdot f}{\pi\cdot 4\cdot V} \Leftrightarrow NF=\frac{A\cdot f}{\pi\cdot V}\ (3)$$

Where: *NF* is the Near Field, *A* is the area of the circle or ring, *D* is the diameter of the transducer, *f* is the frequency of the transducer and *V* is the velocity of the medium.
Equation 1 and 2 can be applicable only for aperture configurations since the combination of the center element with element rings acts approximately as a circular transducer. On the other hand, Equation 3 can be used for individual elements as well as for aperture configurations.

Equation 3 can be further modified in order to include the ring diameters instead of the area. The modified equation replaces the area of a ring with the equivalent relation of the external and internal diameter, as follows;

$$A_{ring} = \pi \cdot \frac{{D_{ex}}^2}{4} - \pi \cdot \frac{{D_{in}}^2}{4} = \frac{\pi}{4}\left({D_{ex}}^2 - {D_{in}}^2\right)$$

Where: A_{ring} is the area of the ring, D_{ex} is the external diameter of the ring and D_{in} is the internal diameter of the ring. Therefore Equation 3 becomes:

$$NF = \frac{A_{ring} \cdot f}{\pi \cdot V} \Leftrightarrow NF = \frac{\pi\left({D_{ex}}^2 - {D_{in}}^2\right) \cdot f}{4 \cdot \pi \cdot V} \Leftrightarrow$$

$$NF = \frac{\left({D_{ex}}^2 - {D_{in}}^2\right) \cdot f}{4 \cdot V} \ (4) \ or \ NF = \frac{{D_{ex}}^2 - {D_{in}}^2}{4 \cdot \lambda} \ (5)$$

Where: *NF* is the Near Field, A_{ring} is the area of the ring, D_{ex} is the external diameter of the ring, D_{in} is the internal diameter of the ring, *f* is the frequency of the transducer, *V* is the velocity of the medium and λ is the wavelength.

2.3 Probe configurations and methodology

The probe configurations are presented for the two probes that are used for the theoretical work. The 2.5MHz probe has 14 elements, 0.2mm kerf, centre disc element of 12mm diameter and external diameter of 48.48mm. The 5MHz probe has 16 elements, same kerf of 0.2mm, centre disc element of 8mm diameter and external diameter of 36.18mm. Except for the centre element, all the other elements are rings. Both probes have been designed by using the equal-area technique which means that all the elements of the probe have approximately the same area. In order to keep the area the same for all the elements, the width of the elements is decreased as the diameter of the ring is increased. Subsequently, each element has a different width. Table 1 and Table 2 demonstrate the specifications for 2.5MHz and 5MHz, respectively.

Table 1. Probe specifications for 2.5MHz probe

Ring No.	Internal diameter (mm)	External diameter (mm)	Element width (mm)
1	0	12	12
2	12.4	17.26	2.43
3	17.66	21.35	1.845
4	21.75	24.84	1.545
5	25.24	27.95	1.355
6	28.35	30.78	1.215
7	31.18	33.41	1.115
8	33.81	35.88	1.035
9	36.28	38.21	0.965
10	38.61	40.43	0.91
11	40.83	42.56	0.865
12	42.96	44.6	0.82
13	45	46.58	0.79
14	46.98	48.48	0.75

Table 2. Probe specifications for 5MHz probe

Ring No.	Internal diameter (mm)	External diameter (mm)	Element width (mm)
1	0	8	8
2	8.4	11.6	1.6
3	12	14.42	1.21
4	14.82	16.84	1.01
5	17.24	19.01	0.885
6	19.41	20.99	0.79
7	21.39	22.84	0.725
8	23.24	24.58	0.67
9	24.98	26.23	0.625
10	26.63	27.8	0.585
11	28.2	29.32	0.56
12	29.72	30.77	0.525
13	31.17	32.18	0.505
14	32.58	33.55	0.485
15	33.95	34.88	0.465
16	35.28	36.18	0.45

Furthermore, in order demonstrate the structure and the width of the elements, the images of the two probes are presented in Figure 2 (the figures have been extracted from Civa). The images are scaled in order to compare the sizes of the two probes.

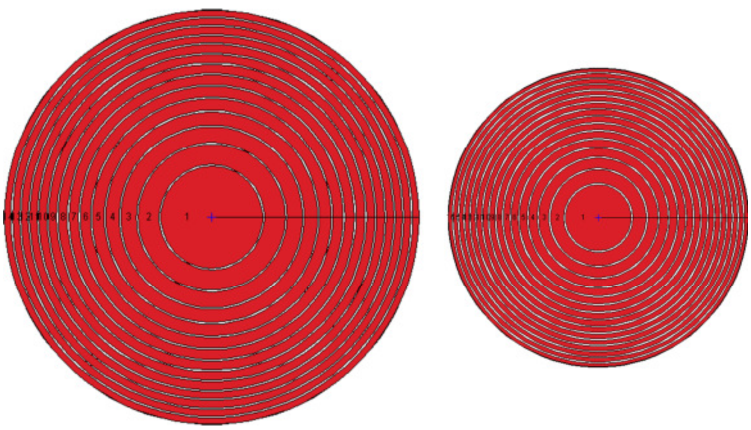

Figure 2. Probe images for 2.5MHz (left) and 5MHz (right)

Near field calculations have been carried out using Equations 2 and 3. It is worth mentioning that Equations 4 and 5 gives exactly the same results as Equation 3. The calculation parameters that were used are presented in Table 3.

Table 3. Near field calculation parameters

Velocity of water (m/s)	1480
Velocity of titanium (m/s)	6100
Center frequency of 2.5MHz probe (MHz)	2.39
Center frequency of 5MHz probe (MHz)	4.4

Furthermore, in order to validate the calculated values, some computational work has been carried out with Civa modelling software. The near field values have been extracted from the on-axis beam propagation plots for individual element. The end of the near field is considered the point in which the highest energy occurs. Moreover, it is possible to reconstruct the beam propagation profile with the equivalent tool which provides useful qualitative results.

2.4 Results

The calculated near field values using the two different approaches for the 2.5MHz probe are presented graphically in Figure 3 and for the 5MHz probe in Figure 4. For individual elements, the near field values are the same for each probe since the probes used the equal-area manufacturing technique. However, small differences are present and for this reason the average values are presented below, both calculated and computational:

- $2.5MHz: 58.12 \pm 0.14mm\ (calculation)$
- $5MHz: 47.58 \pm 0.16mm\ (calculation)$
- $2.5MHz: 60.07 \pm 2.43mm\ (modelling)$
- $5MHz: 48.88 \pm 2.39mm\ (modelling)$

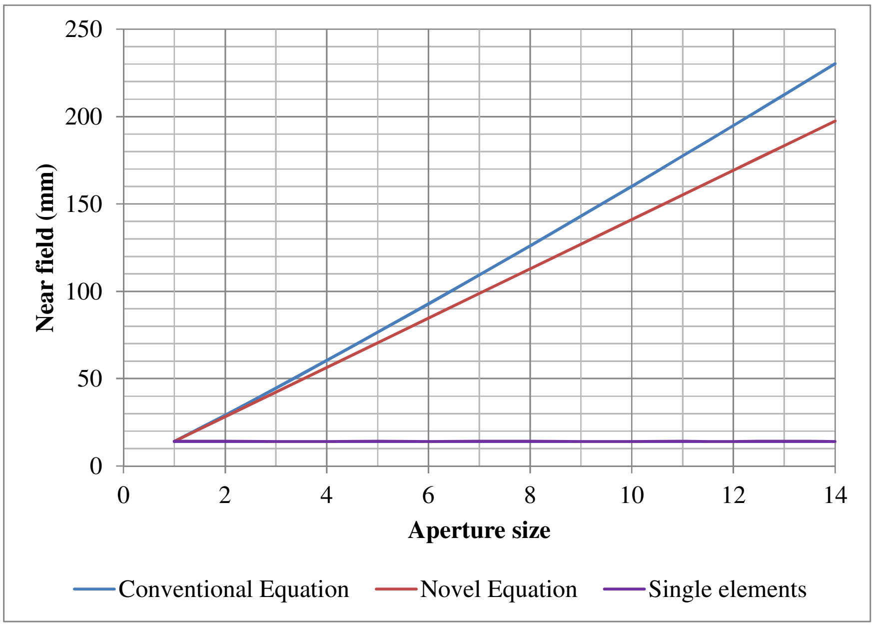

Figure 3. Graphic representation of near field of 2.5MHz probe

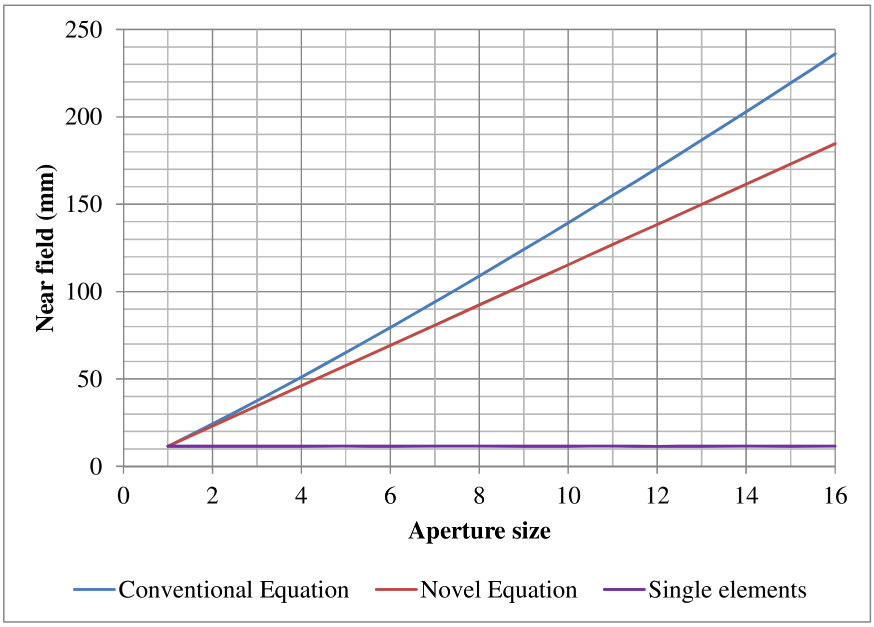

Figure 4. Graphic representation of near field of 5MHz probe

Furthermore, some examples of on-axis beam propagation plots are demonstrated in Figure 5 for the 2.5MHz probe. Beam propagation profiles of each individual element for the 5MHz probe are also presented in Figure 6.

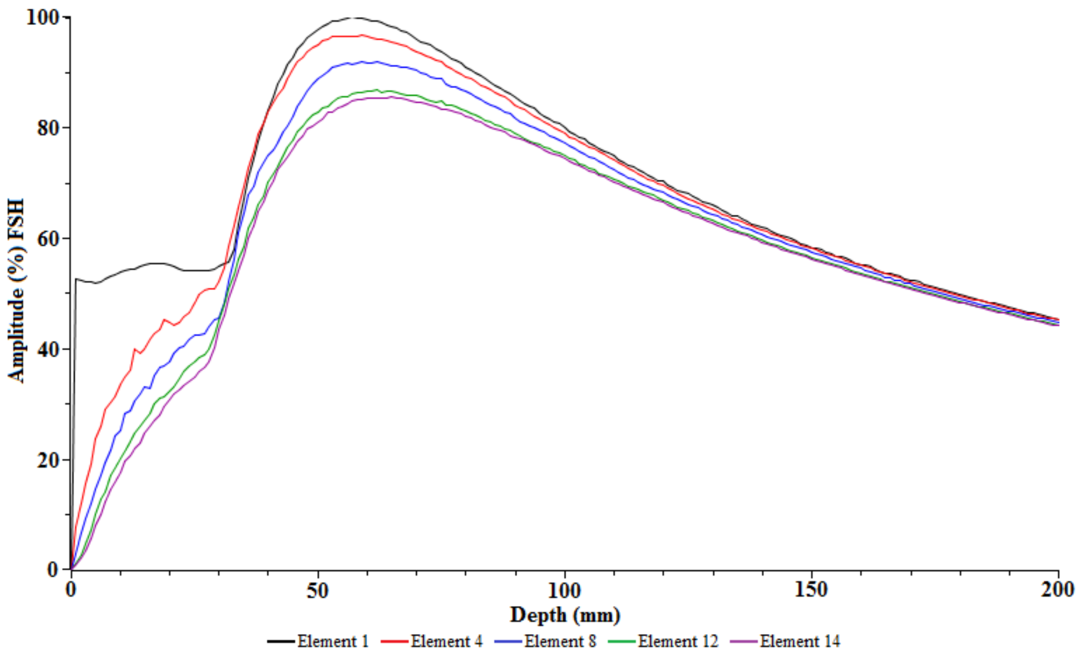

Figure 5. On axis beam propagation plot for some of the elements (2.5MHz)

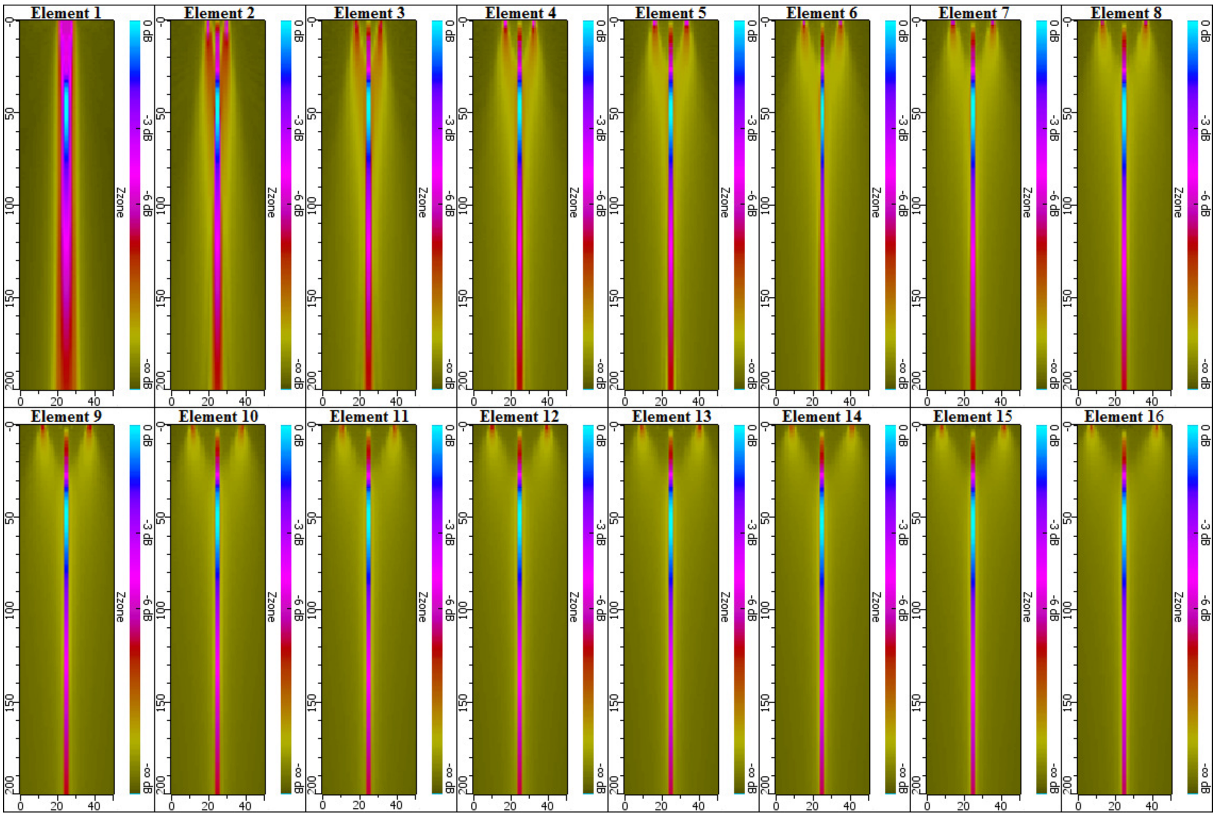

Figure 6. Beam propagation profiles of each element for 5MHz probe

3. Discussion

The first observation that can be made is that the calculated near field values when using an aperture of the probe are not the same when using the two different equations. This is mainly because Equation 2, which uses the diameter of the aperture, includes the kerf distances between the elements. On the contrary, Equation 3, which uses the area of each element, is excluding all the kerfs. Therefore, the near field values when the area is used tend to be smaller. Furthermore, when a small aperture number is used the differences are smaller when compared to a larger aperture number because less kerf distances are taken into account for the calculation. This is noticeable in both probe configurations. For example, for the 2.5MHz probe the difference for the near field in titanium when a 7 element aperture is used, is 10.61mm but when 14 elements are used the difference is 32.79mm.

Even bigger differences are observed for the 5MHz probe. For an aperture size of 8 elements the difference of the near field in titanium is 16.6mm and for an aperture size of 16 elements the difference is 51.36mm. These bigger differences can be explained due to the 5MHz probe having a smaller diameter and more elements than the 2.5MHz probe. Consequently, since the kerf size remains the same for both probes, the elements become thinner. Therefore the differences of the near field values are higher. This phenomenon is more obvious from Figure 3 and Figure 4. As previously mentioned, when the area is used it is possible to calculate the near field for individual elements. Since both probes use the equal-area technique it is expected that all of the elements have the same near field. Indeed from Figure 3 and Figure 4 it can be seen that all elements have pretty much the same area hence the same near field. The elements of the 2.5MHz probe have a near field distance of 14mm in titanium and 58mm in water while the elements of the 5MHz probe have a near field distance of 11.5mm in titanium and 47.5mm in water.

Some interesting observations can be extracted from the comparison of the two probes. First of all, the area of each element of the 2.5MHz probe is more than twice the area of the elements of the 5MHz probe. However, the 5MHz probe has 16 elements and a higher frequency and therefore the near field does not have such a significant difference. What is really interesting is when Equation 2 is used to calculate the near field for a full aperture, the 5MHz probe (NF=236.05mm) has higher near field distance than the 2.5MHz probe (NF=230.21mm). On the contrary, when Equation 3 is used, the 2.5MHz probe has a larger near field distance (NF=197.42mm) than the 5MHz probe (NF=184.96).

The validation of the novel equation has been carried out successfully with Civa. The average values for individual elements are very close. Also this observation can be made from Figure 5. The end of the near field is approximately at the same point despite the drop in amplitude that occurs as the element width is decreased. A nice demonstration of the beam intensity profiles of each element is provided in Figure 6. It is obvious that the near field distance is at the same point for all of the elements, even for element 1. The modelling results demonstrate the accuracy of the derived equation 3 and prove that the near field distances for annular arrays are dependent upon the element area and not on its outer diameter. Furthermore, it can be observed that the focal spot size is decreased as the element width is increased.

4. Conclusions

The investigation has started from the fundamentals, which was necessary because of the unique geometry of the ring-shaped elements that exist within annular arrays. Near field calculations have concluded the following aspects:

- Revealed the limitations of the existing equation for individual elements for ring shaped elements and introduced a novel near field equation.
- The novel equation is applicable for annular arrays and uses the area of the element or alternatively, the external and internal diameter of the element.
- A comparison of the results from the two equations, for the centre element (disc-shaped) it was proven that the novel equation developed is applicable to other element configurations.
- Comparison between the results of the two equations for apertures has revealed differences, which highlighted the importance of validation of the more accurate equation.
- The near field values have been extracted from modelling have been validated against the near field values using the novel equation for individual elements.

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